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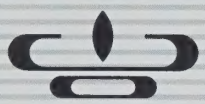

# ontario educational television

# grade 8 geometric concepts; statistics & probability

1966–67
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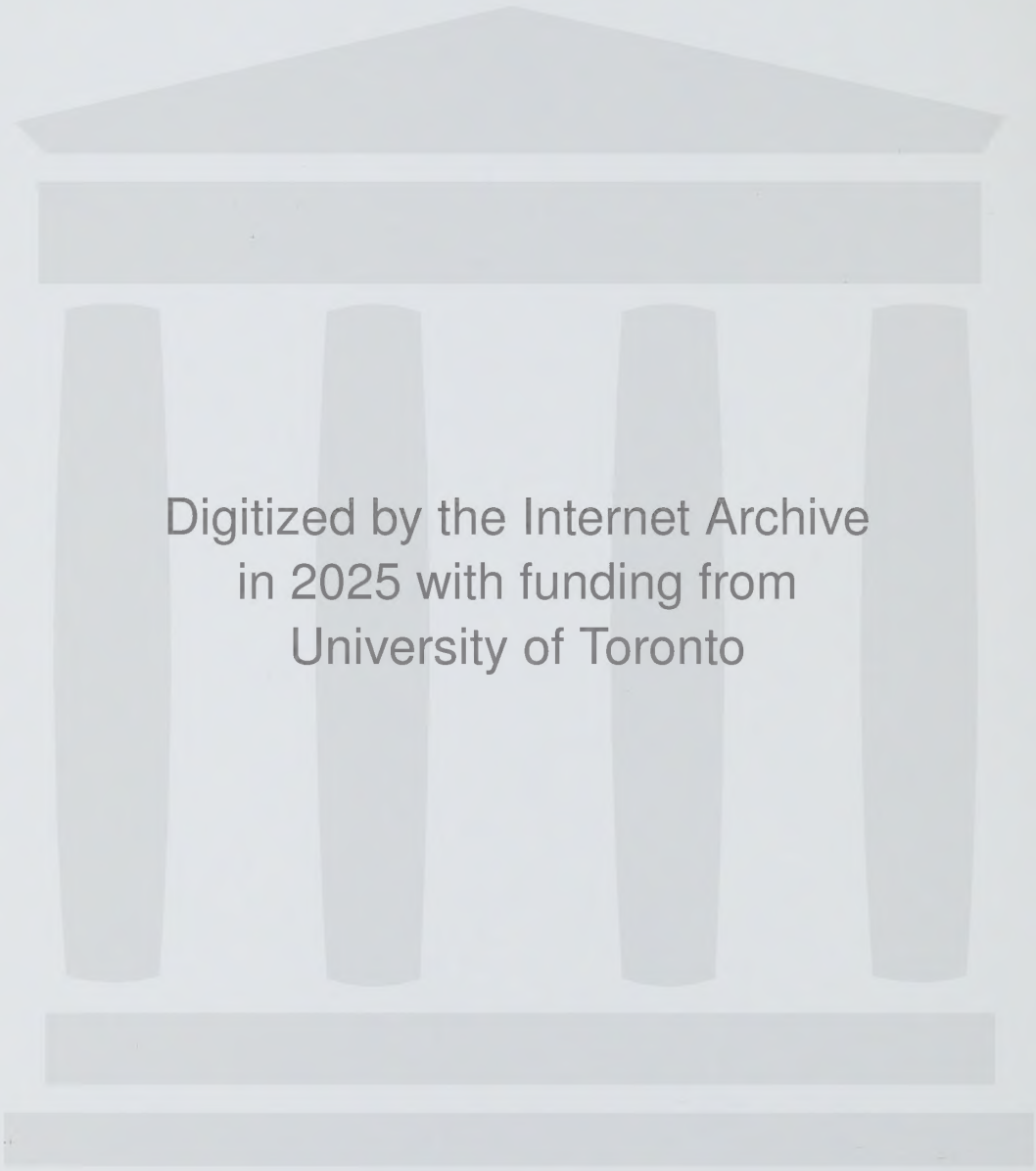
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# Geometric Concepts

GUIDES TO PROGRAMS 14 TO 17
IN SUPPORT OF
GRADE 8 MATHEMATICS

Ontario Educational Television
Ontario Department of Education

Prepared and written by
IVAN ALLEN
Supervisor of Mathematics
North York Board of Education

Subject Supervisors
G. A. KAYE
G. A. SCROGGIE
Assistant Superintendents
Curriculum Division
Department of Education

*For program times, refer to the ETV broadcast schedule*

(For 'Statistics and Probability' series, see Page 7)

# Grade 8 Mathematics
## Geometric Concepts

GENERAL INTRODUCTION

This booklet contains teacher's guide information for the last four programs in the series on *Geometric Concepts*, and for the five programs in the series on *Statistics and Probability*. This constitutes the balance of the guide book to Grade 8 Mathematics television broadcasts, the first half of which was prepared for the first term's programs.

# Program 14
# The Pythagorean Theorem

AIM

This program introduces students to the Pythagorean Theorem and to some simple applications of its results.

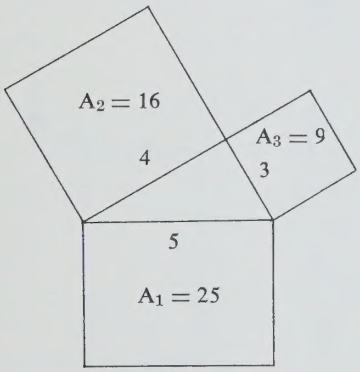
PREPARATION FOR TELECAST

The usual geometric instruments and workbooks are needed for student participation in the program. It would be an advantage if the students had covered the work on square root (Curriculum 1.12B, Unit 3: 7) before watching this program. Construction paper or Bristol board would be useful for follow-up activities.

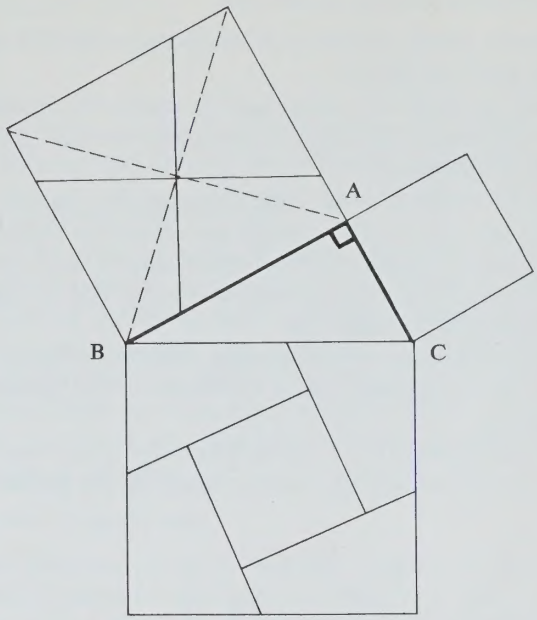
PROGRAM SUMMARY

The program begins as students try to arrange the plastic bars, which were used in previous programs, to form right-angled triangles. They find this is possible if the sides are 3 in., 4 in., and 5 in.; 6 in., 8 in., and 10 in.; 9 in., 12 in., and 15 in.; and 5 in., 12 in., and 13 in. They note the pattern of the first three sets of numbers, but at this time, they do not see any relationship between the numbers in any one set.

Squares are then drawn on the sides of some of the right-angled triangles, as shown below. It is then pointed out in each case that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.




The following example illustrates the Pythagorean Theorem another way. ABC is any right-angled triangle, and squares made from cardboard are placed on each side as shown below. The centre of the square on AB is located by joining the diagonals as indicated by the dotted lines. The other two lines through this centre point are parallel and perpendicular to BC. When the cardboard is cut along these lines, the pieces are rearranged so that the squares on AB and on AC just cover the square on BC as shown.



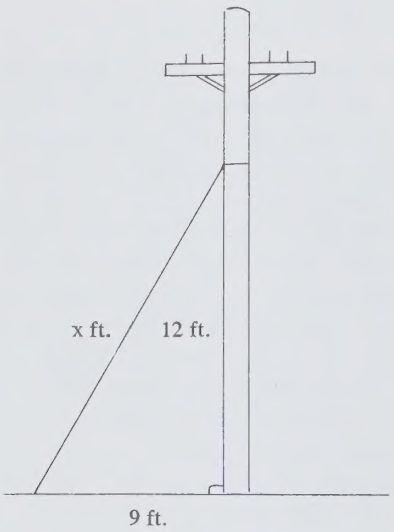

A short historical account of Pythagoras is given. Pythagoras was born about 572 B.C. on the Island of Samos off the coast of Turkey. He founded the Pythagorean Order where the members banded together in a closely knit society with secret rites and observances. They studied philosophy, mathematics and natural science. Their geometry was closely related to arithmetic, and through such a study they actually began the branch of mathematics called 'Number Theory'. Due to political pressures, the order was forced to separate and Pythagoras died, or was murdered, about 507 B.C. While the Pythagorean Theorem is credited to him, his actual method of proof is not known.

The program closes as several questions, using the results of the Pythagorean Theorem, are assigned and solved.

SUGGESTED FOLLOW-UP ACTIVITIES

1. Discuss points of interest in the program and clarify difficulties arising from it.
2. Students could be encouraged to prove the 'Pythagorean Theorem' for any right-angled triangle by a paper-cutting method.
3. Whole numbers which can represent the lengths of the sides of a right-angled triangle are called 'Pythagorean Triples' (e.g. 3, 4, and 5 form a Pythagorean triple since $3^2 + 4^2 = 5^2$). Students could be asked to find as many of these 'triples' as they can.
4. Questions such as the following could be assigned to give the student practice using the results of the theorem.

If a telephone pole is to be braced by a guy-wire as shown in the diagram, find the length of the guy-wire.




# Program 15
# Surface Area of Three-Dimensional Figures

AIM

This program illustrates the finding of surface area by developing methods which the pupils will understand.

PREPARATION FOR TELECAST

The usual geometric instruments and workbooks are needed if the student is to participate fully in the program.

It is also essential that students have a good knowledge of area and perimeter in plane figures before they attempt to find the area of the surfaces of three-dimensional figures.

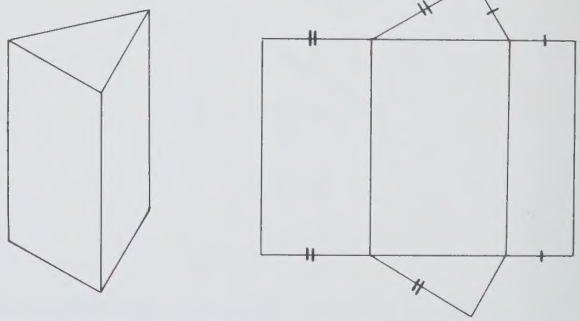
Precut polygons, construction paper, scotch tape and glue would be useful for the follow-up activities.

PROGRAM SUMMARY

The program begins by reviewing the fact that some polygons fit together to cover a region and some do not. Then students illustrate that some polygons can be fitted together to form polyhedrons. Our attention is then turned to the surface area of polyhedrons and we begin by analyzing and calculating the total surface area of a cube.

The next form to be dealt with is the 'rectangular prism' or 'cuboid'. It is unfolded in two slightly different ways and the area calculated in each.

The area of the surface of the triangular prism is found by unfolding it as shown below, and then considering the sum of the areas of the two triangular bases and of



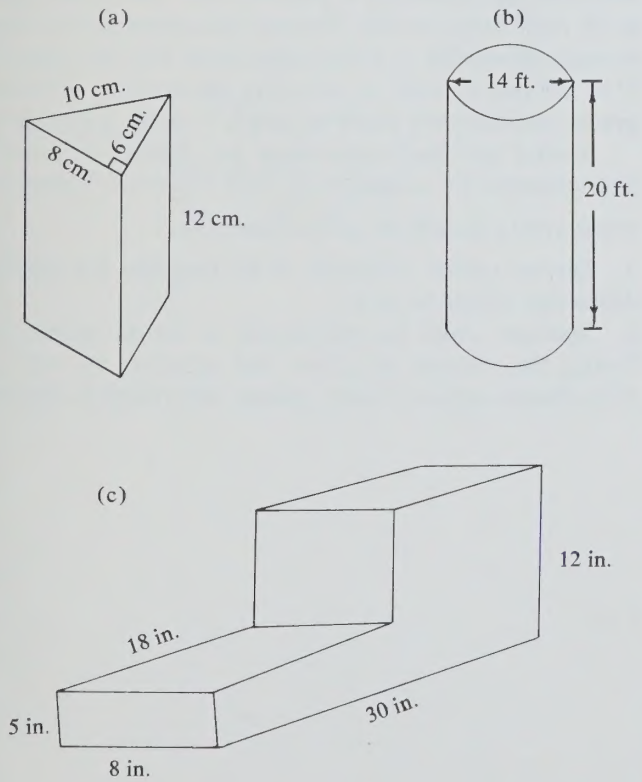
the large rectangle which forms the lateral surfaces. Note that the length of this large rectangle is the same as the perimeter of the triangles.

The last figure to be considered is the cylinder. It is pointed out that the total surface area is found by taking the sum of the areas of the circular bases and the large rectangle which forms its lateral surface. Again, it is observed that the length of this rectangle is the perimeter of the base, which in this case is the circumference of a circle.

Throughout the program, students are asked to participate in calculating areas of different figures.

SUGGESTED FOLLOW-UP ACTIVITIES

1. Discuss points of interest and clarify difficulties which have arisen during the program. It is most helpful during such discussion if students can handle and see solid models.
2. Construction materials could be made available so that students could design and make different models. The regular polyhedrons could be made by taping precut polygons together, or by leaving tabs on the polygons and then gluing them together with the tabs inside.
3. Students could be asked to make a cone from construction paper. (Note that the lateral surface of a cone unfolds to form a sector of a circle.)
4. For additional practice, students could be asked to find the surface area of figures such as the following:

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## Program 16
## Introduction to Volume

AIM

This program is designed to give meaning to the concept of volume, and to introduce methods of calculating it.

PREPARATION FOR TELECAST

Students need the usual geometric instruments and workbooks in order to participate in the program. If full-sized models of the cubic inch, cubic foot, cubic yard, cubic centimetre and litre are available, they will help the student to have a practical knowledge of these units. Such commodities as water, sand, wooden cubes, and containers of different sizes and shapes would be useful for the follow-up activities.

PROGRAM SUMMARY

The program opens as students compare the size of a 'pop' bottle and a 'pop' can by finding out which will hold the most water. Then they compare the volume of two rocks by finding which displaces the most water. In order to find out how much larger a cubical container is than a water pitcher, a 'glassful' is defined as an arbitrary unit of volume. Then the volume of each container is measured in terms of this unit.

After discussing the need for standard units, the cubic inch is defined as the space occupied by a cube with each edge one inch in length. Similarly the cubic centimetre is defined as the space occupied by a cube with each edge one centimetre in length. Then other units are introduced and the following relationships established:

British System

1 cubic yard (cu. yd.) = 27 cubic feet (cu. ft.)
1 cubic foot (cu. ft.) = 1728 cubic inches (cu. in.)

Metric System

1 litre (1) = 1000 cubic centimetres (cu.cm.)
1 litre = 1000 millilitres (ml.)

With the use of one-inch cubes, students then develop the formula $V = lwh$ for finding the volume of a rectangular solid. Note 'V' represents the volume, 'l' represents length, 'w' represents width, and 'h' represents height.

The program closes as several questions involving volume are assigned and the solutions are discussed.

SUGGESTED FOLLOW-UP ACTIVITIES

1. Discuss points of interest in the program and clarify any difficulties arising from it.
2. Students could be encouraged to estimate the volume of different objects, and then be given the opportunity to devise methods of comparing, measuring and calculating their volumes. Water, sand and wooden cubes may serve as useful aids in developing fundamental ideas associated with volume.
3. Questions such as the following could be assigned in order to give students practice in calculating volume:

a) Find the volume of water required to fill the container illustrated in Figure 1. Give your answer to the nearest litre.
b) Find the volume of the wooden model illustrated in Figure 2. Give your answer to the nearest cubic inch.

Figure 1

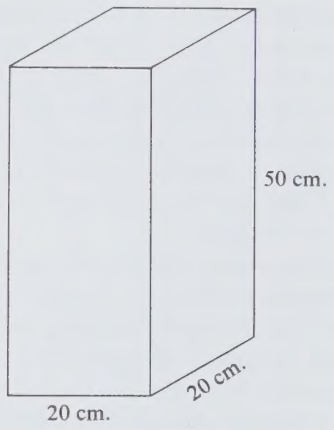
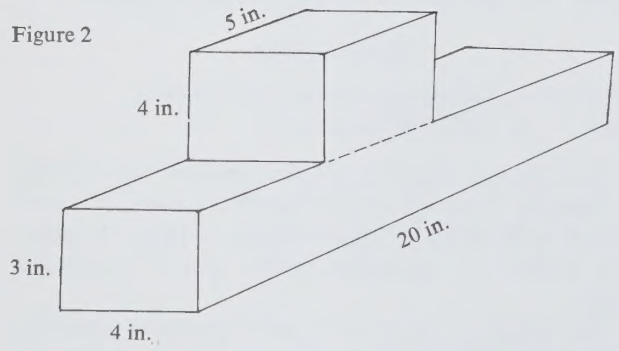



Figure 2




## Program 17
## Volume of Prisms and Cylinders

AIM

With the use of concrete materials, this program develops the idea that the volume of a right prism and cylinder may be found by multiplying the area of the base by the respective height.

PREPARATION FOR TELECAST

The usual geometric instruments and workbooks are needed to assure student participation in the program. The concrete materials which were suggested for Program 16 will also be useful for the follow-up activities in this program.

PROGRAM SUMMARY

The program opens by reviewing the formula ($V = 1wh$) for finding the volume of a rectangular prism. However, by observing models of rectangular prisms made from inch cubes, it is noted that the volume can also be found by multiplying the area of the base by the height. Thus a formula $V = Bh$ is established for finding the volume of a rectangular solid, where 'V' represents volume, 'B' represents area of each base and 'h' represents the height. (Note that the height is the perpendicular distance between the two bases of a prism.) In this case the formulae $V = 1wh$ and $V = Bh$ are comparable, since '1w' in the first could be considered to represent the area of the base, as 'B' does in the second. Through discussion and the use of concrete models, it is then established that the formula $V = Bh$ can be used to calculate the volume of other prisms and cylinders where no length or width is involved.

Several questions concerning the volume of prisms and cylinders are assigned and their solutions discussed.

SUGGESTED FOLLOW-UP ACTIVITIES

1. Discuss points of interest in the program and clarify difficulties arising from it.
2. Students could be encouraged to devise means of finding the volumes of prisms and cylinders as well as other shapes, such as cones, spheres and irregular objects.

3. Questions such as the following could be assigned in order to give students practice in calculating volume:

a) Find the volume of the water required to fill a circular swimming pool to a height of 3 feet if the diameter of the pool is 10 feet. Give your answer to the nearest cubic foot.

b) Find the volume of the triangular prism which is illustrated in Figure 1. Give your answer to the nearest cubic centimetre.

c) The diagram in Figure 2 represents a steel tank on a water truck. Find, to the nearest gallon, how many gallons of water it will hold if each cubic foot equals 6¼ gallons.

Figure 1

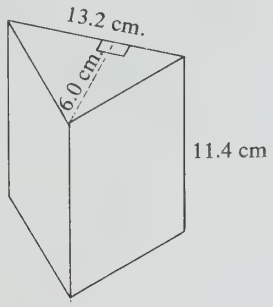
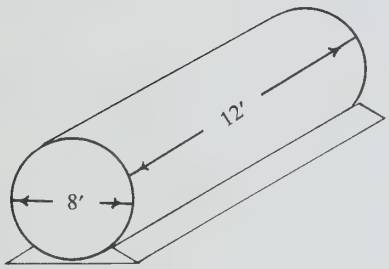
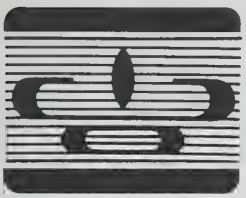



Figure 2



# Statistics and Probability

A SERIES OF FIVE PROGRAMS
IN SUPPORT OF
GRADE 8 MATHEMATICS



Ontario Educational Television
Ontario Department of Education

Prepared and written by
JAMES B. BLUE
Principal
Western School
Windsor

Subject Supervisors
G. A. KAYE
G. A. SCROGGIE
Assistant Superintendents
Curriculum Division
Department of Education

*For program times, refer to the ETV broadcast schedule*

## Statistics and Probability

GENERAL INTRODUCTION TO PROGRAMS 18 TO 22

This series of five programs forms a unit, separate in thought from the previous programs on geometric concepts. It is based on Unit 7 of the Course of Study for Grade 8 Mathematics (see Curriculum 1.12B: An Introduction to some of the Language used in the Presentation and Interpretation of Data).

The guides for this section contain exercises in both preparation and follow-up for the programs. It is essential that the students have ready the materials indicated, as they will need them to participate during certain parts of the telecasts.

Statistics and probability both have a wide application in our scientific world. It is increasingly important that the students of today be given the chance to acquire some knowledge in this field of Mathematics.

# Program 18
# Statistics

AIM

The study of statistics is introduced in this program, which presents data through the pictograph and the bar graph.

SUGGESTED ACTIVITIES BEFORE TELECAST

1. A class discussion of the statistics which confront us daily in the newspapers, magazines, and other forms of communication would be useful.
2. A review of 'rounding off' numbers might be helpful.
3. Before the telecast, the pupils might be asked the following question which will be answered on the program: In what ways do we collect and use data?

PREPARATION FOR TELECAST

Each pupil will require the following materials: pencil and ruler, two sheets of graph paper, and two sheets of blank paper. It would be advisable for the teacher to tell the pupils in advance that they will be drawing a bar graph during the program.

PROGRAM SUMMARY

The program opens with four students analyzing their newspapers. They are all finding that numbers are used in reporting the news of their interest. Each student discovers where the data of his interest originated: one of the boys takes us to a football field where he interviews the statistician; one of the girls takes us to a meteorological institute where the weather facts originate.

Data can be presented in many ways. In this program, pictographs are explained as a useful means of picturing numerical relationships. Bar graphs are introduced and we see how they clearly and quickly summarize data relationships.

Pupils should become aware of the wide application of statistics in our modern scientific age.

SUGGESTED FOLLOW-UP ACTIVITIES

1. Data for the construction of a bar graph is shown on the program. You may wish to copy this data on the board if the students have not been able to complete copying it from the screen.

*Data for the bar graph*

Title: Population of five Ontario Metropolitan Areas (Census 1961)

| CITY | POPULATION |
|---|---|
| Hamilton | 365,189 |
| Kitchener | 153,864 |
| Windsor | 193,365 |
| Sudbury | 110,694 |
| Ottawa | 429,750 |

Suggestion: Round off population figures to the nearest ten thousand.

2. Another suggested exercise is the construction of a pictograph.

*Data for pictograph*

Title: Dropouts Through the Grades

| GRADE | YEAR | NUMBER |
|---|---|---|
| 9 | 1961 | 100 |
| 10 | 1962 | 83 |
| 11 | 1963 | 62 |
| 12 | 1964 | 57 |
| 13 | 1965 | 33 |
| University | 1966 | 16 |

Suggestion: Have the pupils use a stick symbol to represent a given number of students.

3. The students will be encouraged to find examples of data, pictographs, and bar graphs in magazines and newspapers to make a classroom display.

# Program 19
# Statistics

AIM

The study of statistics is further developed with the demonstration of how to draw and use a circle graph, a frequency distribution, a histogram, and a line graph.

SUGGESTED ACTIVITIES BEFORE TELECAST

1. As an introduction to the construction of circle graphs:
   a) Review the use of the protractor to construct angles of given sizes.
   b) Review the measurement of an angle formed by two radii of a circle.
2. Review line graphs showing rate and comparison.

PREPARATION FOR TELECAST

Each pupil will require the following materials: pencil and ruler, two sheets of blank paper, two sheets of graph paper, and the completed assignment from Program 18.

PROGRAM SUMMARY

At the beginning of the program the bar graph that the students were left to draw as a follow-up for Program 18 is shown. The rectangular distribution graph leads to a detailed outline of how to make a circle graph, accompanied by a few examples of its use.

Frequency distribution is clarified by familiar examples, such as counting the frequency of different types of vehicles in traffic. Tallies are used to determine the frequency, and this data is organized in frequency distribution tables. Histograms are developed from these tables: mark intervals and group boundaries are discussed in this context.

Several types of line graphs exist. Among those discussed on the program are the frequency polygon, and straight line graphs to show rate and comparison.

| MARK INTERVAL | TALLY | FREQUENCY |
|---|---|---|
| 28.5 to 30.5 | // | 2 |
| 26.5 to 28.5 | 𝍸 | 5 |
| 24.5 to 26.5 | 𝍸 𝍸 \| | 11 |
| 22.5 to 24.5 | 𝍸 //// | 9 |
| 20.5 to 22.5 | 𝍸 | 5 |
| 18.5 to 20.5 | /// | 3 |

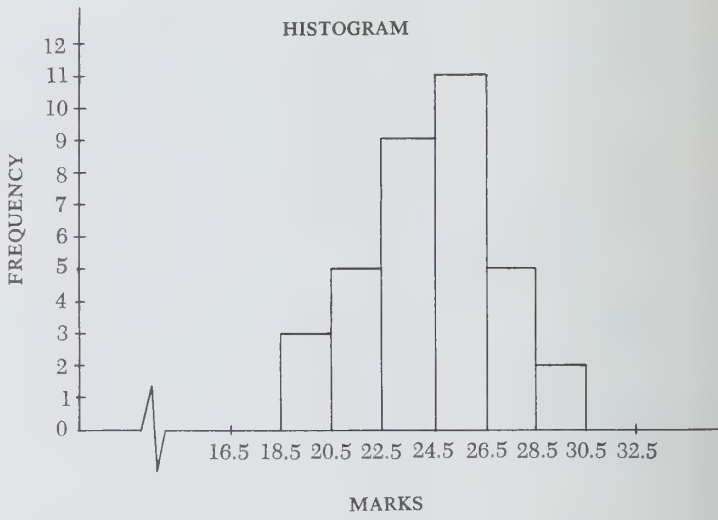
SUGGESTED FOLLOW-UP ACTIVITIES

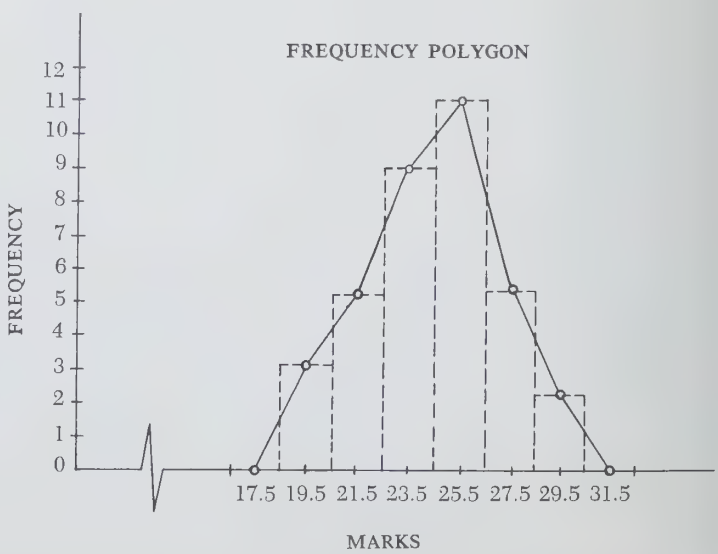
1. On the program, the pupils will be given 35 test scores, from which they are requested to construct a combined histogram and frequency polygon for the distribution. Unfortunately, time does not allow these numbers to be held on the screen long enough for all pupils to copy them. We suggest that the teacher copy the list of figures on the board following the program, as well as the accompanying instructions.

DATA: 35 TEST SCORES

| | | | | | | |
|---|---|---|---|---|---|---|
| 21 | 26 | 21 | 20 | 23 | 24 | 22 |
| 19 | 24 | 26 | 25 | 23 | 26 | 29 |
| 21 | 24 | 19 | 25 | 26 | 25 | 22 |
| 25 | 27 | 23 | 26 | 24 | 25 | 30 |
| 25 | 23 | 27 | 24 | 28 | 28 | 28 |

The following diagrams illustrate the answers which will also be shown at the beginning of Program 20.



## Program 20
## Statistics: Measures of Central Tendency

AIM

This program familiarizes the student with three different measures of central tendency: the arithmetic mean, the median, and the mode.

SUGGESTED ACTIVITIES BEFORE TELECAST

1. The class might discuss the meaning of the word 'average' as it exists in the minds of the pupils.
2. Review briefly the frequency distribution chart, and the following graphs: pictograph, bar graph, circle graph, histogram, and line graph.

PREPARATION FOR TELECAST

Each student will require the following materials: pencil and ruler, two sheets of blank paper, and the completed assignment from Program 19.

PROGRAM SUMMARY

The program begins with the answer to the question given to the class at the end of Program 19. Following this is a brief review of the various types of graphs which have been studied in the previous two programs.

'Average' is introduced as another way of summarizing a set of data, in this case, by means of a single score. Different meanings of the word 'average' are discussed; experiments and data which are of interest to the students illustrate the true meanings of mode, median, and arithmetic mean.

SUGGESTED FOLLOW-UP ACTIVITIES

1. The program presents the following numbers for the students to analyze after the program. They are asked to find the mode, the median, and the mean of the numbers: 2, 3, 4, 7, 5, 7, 3, 3, 10, 4, 9. The answers to this question will not be dealt with on Program 21. They are included here for classroom use.

a) Mode: 3 (because it appears most often or has the *greatest* frequency).

b) Median: First the numbers must be arranged in order: 2, 3, 3, 3, 4, 4, 5, 7, 7, 9, 10.
The median is 4.

c) Mean: First the total must be calculated, and then divided by the number of numbers involved:

$$\frac{2+3+4+7+5+7+3+3+10+4+9}{11}$$

$$= \frac{57}{11} = 5\tfrac{2}{11}$$

The mean is $5\frac{2}{11}$.

2. Another exercise in central tendency is suggested. Find the mode, median and mean of the following data: 3, 3, 1, 2, 2, 3, 2, 4, 5, 4, 3, 4, 2, 3, 3, 4.

Answers:

a) Mode 3
b) Median 3
c) Mean 3

## Program 21
## Probability

AIM

This program endeavours to develop the concept of probability as the ratio of the number of favourable cases to the total number of cases.

$$\text{Probability} = \frac{\text{number of favourable cases}}{\text{total number of cases}}$$

SUGGESTED ACTIVITIES BEFORE TELECAST

1. Review briefly the unit of Curriculum I. 12B dealing with Ratios.
2. Review the five regular solids that were presented on a previous telecast. These are the tetrahedron, hexahedron, octahedron, dodecahedron and icosahedron.
3. Practise in reducing fractions to their simplest form.

PREPARATION FOR TELECAST

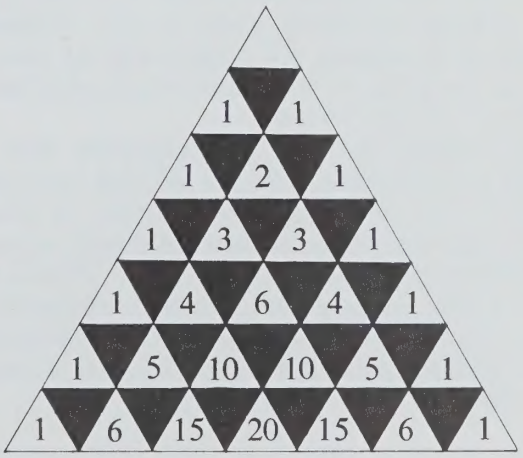
Each pupil will require the following materials: pencil, ruler and two sheets of blank paper.

PROGRAM SUMMARY

The familiarity of 'probability' statements in conversation is illustrated in the introduction to this program. Probabilities are calculated in simple cases where the number of favourable and total cases can easily be discovered, for example: the probability of getting a head when flipping a coin, or a five when rolling a die.

Probability charts list the possible outcome when one flips two coins, three coins, and so on. These charts introduce 'Pascal's Triangle', which is used to determine probabilities.

DIAGRAM OF PASCAL'S TRIANGLE




The probability board is another method of predicting probabilities.

SUGGESTED FOLLOW-UP ACTIVITIES

1. The class might construct a probability board.
2. Discuss all the possible outcomes when two dice are rolled, and have the students set up a chart of these.
3. Pupils might write biographical sketches of famous mathematicians such as Blaise Pascal and Pierre LaPlace.
4. Discuss the probabilities of each of the following events:

a) A family of 3 children being all boys.
b) Tossing the sum of 7 with 2 dice, 3 dice, etc.

## Program 22
## Probability

AIM

Probability is presented in this program as an exciting and rapidly growing branch of Mathematics.

SUGGESTED ACTIVITIES BEFORE TELECAST

1. Multiplication of decimal fractions might be reviewed.
2. Review division by a common fraction using the reciprocal method.
3. The students should be familiar with Probability Ratio, which was introduced on Program 21.

$$\text{Probability} = \frac{\text{number of favourable cases}}{\text{total number of possible cases}}$$

PREPARATION FOR TELECAST

Each pupil will require the following materials: pencil and ruler, two sheets of blank paper.

PROGRAM SUMMARY

The program begins with a brief review of Program 21.

In studying probability, when the total number of cases cannot be counted, one is faced with the 'need for sampling'. An instance of this occurs in television ratings, which are based on a sample.

The meaning of 'odds' is distinguished from the meaning of 'probability'. The law of large numbers is introduced; this states that as the number of trials is increased, the closer the calculated probability comes to the Mathematical Probability Ratio. This is illustrated by tossing a coin *many* times. Empirical Probability is explained as it is used in industry, commerce and sports.

The program closes with 'doop'—the game of predicting 'doubles'.

FOLLOW-UP ACTIVITIES

1. Have each pupil in the class flip a coin ten times. Record the combined results, and calculate the probability of a 'head'. Compare this with the mathematical probability of $\frac{1}{2}$.
2. A bowl contains three red, two blue, three yellow and two green marbles. A single marble is drawn from the bowl.

a) What is the probability that it is red?
b) What are the odds in favour of its being red?
c) What is the probability that it is blue or green?
d) What are the odds against its being blue or green?
e) What is the probability that it is yellow?

The answers are calculated as follows:

a) $P(\text{red}) = \frac{3}{10}$

b) Odds in favour of red $= \dfrac{\frac{3}{10}}{\frac{7}{10}} = \dfrac{\frac{3}{10} \times \frac{10}{7}}{\frac{7}{10} \times \frac{10}{7}} = \frac{3}{7}$

c) $P(\text{blue or green}) = \frac{4}{10}$ or $\frac{2}{5}$

d) Odds against blue or green

$$= \frac{\frac{6}{10}}{\frac{4}{10}} = \frac{\frac{6}{10} \times \frac{10}{4}}{\frac{4}{10} \times \frac{10}{4}} = \frac{6}{4} = \frac{3}{2}$$

e) $P(\text{yellow}) = \frac{3}{10}$